Grade 8 Fraction Division

8.N.6	
Demonstrate an understanding of multiplying and dividing positive fractions and mixed numbers, concretely, pictorially, and symbolically.	1. Identify the operation required to solve a problem involving positive fractions. [DEVELOPED] 2. Provide a context that requires the multiplying of two positive fractions. [NOT DEVELOPED] 3. Provide a context that requires the dividing of two positive fractions. [DEVELOPED] 4. Express a positive mixed number as an improper fraction and a positive fraction as a mixed number. [NOTE] 5. Model multiplication of a positive fraction by a whole number, concretely or pictorially, and record the process. [NOT DEVELOPED] 6. Model multiplication of a positive fraction by a positive fraction, concretely or pictorially using an area model, and record the process. [NOT DEVELOPED] 7. Model division of a positive proper fraction by a whole number, concretely or pictorially, and record the process. [DEVELOPED] 8. Model pictorially division of a positive proper fraction by a positive proper fraction, and record the process. [DEVELOPED] 9. Generalize and apply rules for multiplying and dividing positive fractions, including mixed numbers. [DEVELOPED only for DIVISION] 10. Solve a problem involving positive fractions taking into consideration order of operations (limited to problems with positive solutions). [DEVELOPED]

Clarification of the outcome:

- ✦ This outcome concerns understanding and being able to use an efficient fraction division algorithm.

Required close-to-at-hand prior knowledge:

- ✤ Understands the part of a whole and measure meanings of fraction.
- ✤ Understands mixed numbers, reciprocals, fraction equivalency, and fraction multiplication.

NOTE: (Achievement indicator #4)

In grade 7, students are expected to add/subtract mixed numbers. That involves converting between mixed numbers and improper fractions. The assumption here is that students can convert. Therefore indicator #4 is not addressed.

SET SCENE stage

The problem task to present to students:

- *Bert has 7 sticks of gum. He breaks each stick into half-size pieces to share with his friends. How many half-size pieces will he get?*
- *Bert (same guy) has 1/5 of a pie. He cuts it into two equal pieces. How big is each piece?*
- *Bert (still same guy) has 1/2 of a blueberry pie in the fridge. He makes 1/6 size pieces for his friends to eat. How many 1/6 size pieces will he get?*

Organize students into groups. Ask each group to solve the problems in whatever way makes sense to them. Allow them freedom to explore solution paths.

Comments:

The purpose of the task is present a situation that begins students' journey into fraction division.

DEVELOP stage

Activity 1: Revisits SET SCENE

Ask selected groups to present their methods of solving and their solutions for the SET SCENE problems. Do not indicate correct/not correct. Just acknowledge the approaches.

Guide them to interpret both problems in a division way ($7 \div 1/2 = ?$; $1/5 \div 2$; $1/2 \div 1/6 = ?$). This may be more difficult than you might imagine because the questions are not simply about whole numbers. The best explanation for why the problems involve division is to revisit the meaning of divisions as splitting up into equal groups and then discuss whether splitting up into equal groups is involved in the problems.

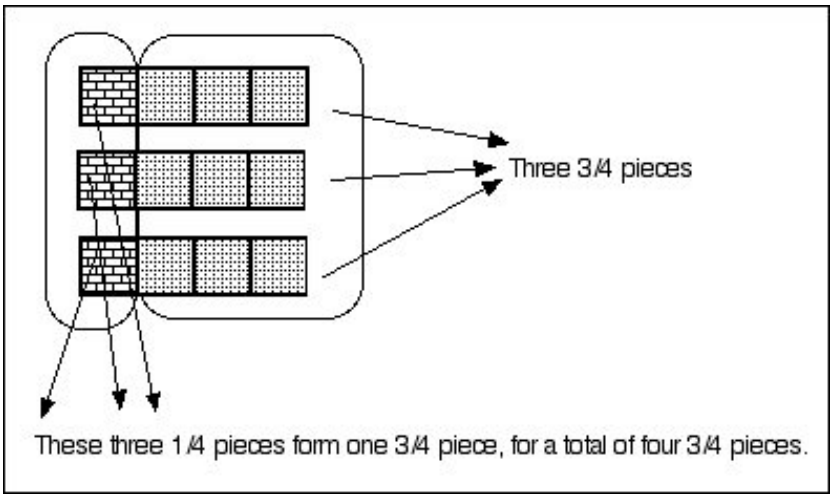
Mention that students will first consider a whole number ÷ a fraction and a fraction ÷ a whole number before considering a fraction ÷ a fraction.

Activity 2: Addresses achievement indicators 1 and 3 (whole ÷ unit fraction).

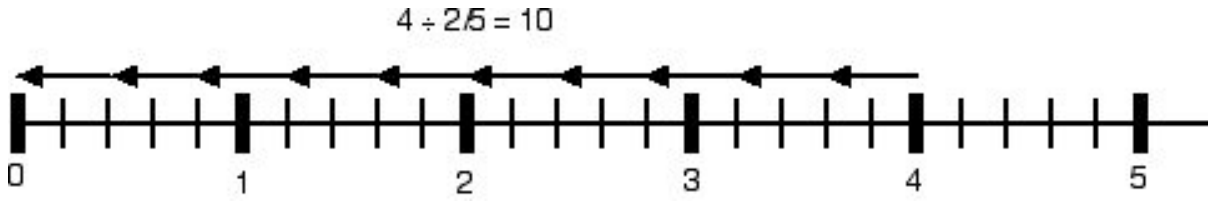
- Pose the problem: "*Mary has 5 chocolate bars. She wants to share her bars equally with her friends. Mary cuts each chocolate bar up into thirds. How many friends will get a third of a chocolate bar?*" Ask students to express the problem with a number sentence (expect: 5 ÷ 1/3 = 15) and then solve it any way they want (some may draw 5 rectangles, split each rectangle into three pieces, and count the number of pieces obtained). Discuss their solutions. Discuss if the answer to a division always results in a smaller number (it does not).
- Pose the problem: "*I have a 4 cm long string and cut it into strips that each are 1/2 cm long. How many strips will I get?*" Ask students to express the problem with a number sentence (expect: 4 ÷ 1/2 = 8) and then solve it any way they want (some may draw a line 4 cm long, split it into 1/2 cm sections, and count the number of sections obtained). Discuss their solutions. Discuss if the answer to a division always results in a smaller number.
- Write the division sentences from the two problems (5 ÷ 1/3 = 15; 4 ÷ 1/2 = 8) on the board. Ask students to invent a method for doing fraction division for a whole number divided by a unit fraction (1/n). Ensure that they realize one method is to multiply the whole number by the denominator of the fraction. Ask if they think this short cut method will always work. Leave the answer in limbo for the moment.
- Provide four practice questions for a whole ÷ unit fraction (e.g. 3 ÷ 1/6). Ask students to use the short cut method to obtain the answers. Select one of the questions and confirm the answer by using fraction bars or rectangle drawings.

Activity 3: Addresses achievement indicators 1 and 3 (whole ÷ proper fraction).

- Pose the problem: "*Mary has 3 chocolate bars. She wants to share her bars equally with her friends. Mary cuts each chocolate bar into three-quarter size pieces. How many friends will get three-quarters of a chocolate bar?*"
- Provide fraction bars/grid paper. Ask students to express the problem with a number sentence and then use fraction bars/grid paper to solve it (refer to diagram). Discuss the solution (3 ÷ 3/4 = 4).



- Pose the problem: "*I have a 4 cm long string and cut it into strips that each are 2/5 cm long. How many 2/5 cm strips will I get?*" Provide a number line marked off in fifths. Ask students to express the problem with a number sentence and then use the number line to solve it. Expect them to mark off 2/5 long strips and count the number of 2/5 long strips obtained (see diagram). Discuss the solution (4 ÷ 2/5 = 10).



For division, you start at the number being divided and move towards zero.

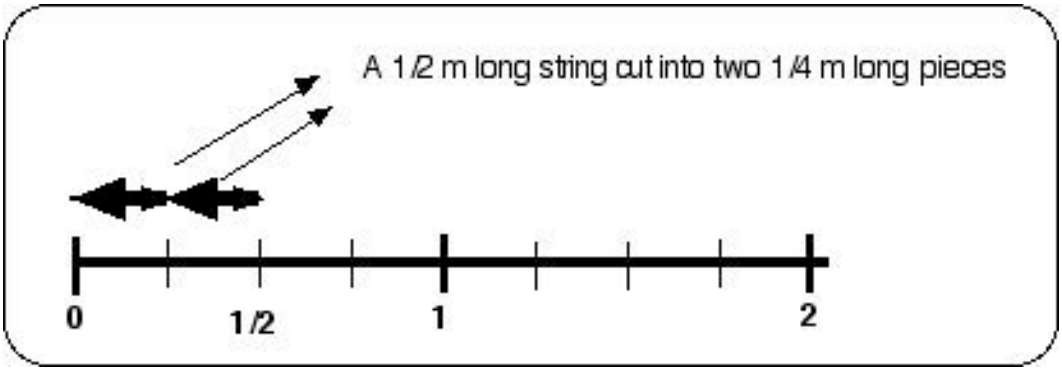
- Write the number sentences (3 ÷ 3/4 = 4; 4 ÷ 2/5 = 10) from the two problems on the board beside the previous two number sentences (5 ÷ 1/3 = 15; 4 ÷ 1/2 = 8). Ask students if they think the short cut method for the previous two number sentences works for the two new number sentences. Expect them to realize that multiplying the whole number times the denominator of the fraction no longer works (e.g. for 4 ÷ 2/5, 5 x 4 does not result in the correct answer of 10). Ask students to invent a different method that works for all the number sentences. Expect them to invent a new method that involves multiplying the whole number by the denominator of the fraction and then dividing that result by the numerator of the fraction (e.g. 4 ÷ 2/5 = 4 x 5 ÷ 2).
- Provide four practice questions for a whole ÷ a proper fraction for which the answer is a whole number (e.g. 10 ÷ 5/3 = 10 x 3 ÷ 5 = 6). Ask students to use the new method to obtain the answers. Select one of the questions and confirm the answer by using a number line.

Activity 4: Addresses achievement indicators 1, 3, and 7 (fraction ÷ whole).

- Pose the problem: "*Bill has 1/2 a chocolate bar. He wants to split it into two equal pieces. What fraction of a whole chocolate bar will each piece be?*" Provide fraction bars/grid paper. Ask students to express the problem with a number sentence (1/2 ÷ 2 = ?) and then use fraction bars/grid paper to solve it. Expect them to make a 1/2 chocolate bar and split it two equal parts, obtaining quarter pieces. Discuss the solution. Ask students if the division method from activity #3 works for a fraction ÷ a whole. Expect doubt. Leave the matter in limbo for the moment.

Activity 5: Addresses achievement indicators 1, 3, 7, 8, and 9 (fraction ÷ fraction).

- Pose the problem: "*I have a string 1/2 m long. I cut it into 1/4 m long strips. How many strips can I make?*" Provide number lines marked off in fourths. Ask students to express the problem with a number sentence (1/2 ÷ 1/4 = ?) and then use the number line to solve it. Expect them to mark off 1/4 long strips and count the number of 1/4 long strips obtained (see diagram). Discuss the solution (1/2 ÷ 1/4 = 2).



Activity 6: Addresses achievement indicators 1 and 9 (fraction ÷ fraction).

- Briefly revisit the concept of a reciprocal or flip of a fraction (e.g. the reciprocal of 2/3 is 3/2). Provide practice as needed.
- Select four fraction division number sentences from the previous activities and write them on the board. One of the number sentences MUST BE the one from activity #5 (1/2 ÷ 1/4 = 2). Beside each division number sentence, write a corresponding number sentence that involves multiplication by the reciprocal. DO NOT include the answers for the multiplication number sentences. Here is a possible list of number sentences.

Division	Multiplication
1/2 ÷ 1/4 = 2	1/2 x 4/1 =
5 ÷ 1/3 = 15	5 x 3/1 =
4 ÷ 2/5 = 10	4 x 5/2 =
1/2 ÷ 2 = 1/4	1/2 x 1/2 =

- Ask students to obtain the answers to the multiplication questions. Ask if they notice that the multiplication answers match the answers to the corresponding division questions. Ask them about the relationship between the divisor in the division question and the second multiplying number in the multiplication questions (e.g row 1: between 1/4 and 4/1). Ensure students realize that the numbers are reciprocals.
- Ask students if they see a FAST METHOD for dividing fractions. Ensure they realize fraction division is simply multiplication by the reciprocal (e.g. 6 ÷ 4/3 = 6 x 3/4; e.g. 4/5 ÷ 1/7 = 4/5 x 7/1).

Activity 7: Addresses achievement indicator 9 and practice.

- Provide students with three division questions that concern a proper fraction ÷ a proper fraction and two each of a fraction ÷ whole and a whole ÷ a fraction. Ask students to use the FAST method to obtain answers. Assist and discuss as needed.

Activity 8: Addresses achievement indicator 9 and revisits SET SCENE.

- Revisit the three problems from SET SCENE. Ask students to redo them using what they have learned to this point. Discuss their work.

Activity 9: Addresses achievement indicators 1, 4, and 9.

- When you are confident that students can divide fractions using the FAST METHOD, refresh their memories about converting mixed numbers to improper fractions. Provide practice as needed.
- Pose a problem involving a mixed number ÷ a whole number. (e.g. Y*ou buy 1 3/4 kg of butter. You separate the butter into three equal portions. How much butter is in each portion?)* Ask students to represent the problem using a division number sentence. Expect 1 3/4 ÷ 3 = ?). Ask them to change the mixed number to a fraction greater than 1 and to obtain an answer using the FAST METHOD for dividing fractions.
- Provide six other questions involving mixed numbers, proper fractions, and whole numbers. Ask students to obtain the answers. Discuss their work.

Activity 10: Addresses achievement indicator 10.

- Present students with arithmetic tasks that involve addition/subtraction, multiplication, and division of fractions and mixed numbers (e.g. 3 1/2 ÷ 2 + 3/4 x 1/2 + 5/6). Ask them to obtain answers, paying attention to the order of operations. Discuss their work.

Activity 11: Assessment of teaching.

- Provide students with two simple problems (problems, not arithmetic questions), involving a proper fraction ÷ a whole number and one problem involving a mixed number fraction ÷ a proper fraction. Ask students to solve the problems.

If all is well with the assessment of teaching, engage students in PRACTICE (the conclusion to the lesson plan).

An example of a partial well-designed worksheet follows.

The worksheet contains a sampling of question types. More questions of each type are needed.

The MAINTAIN stage follows the sample worksheet.

Question 1.

a) Do the fraction division question, $4 \div 2/3$, by repeatedly subtracting 2/3 and counting how many times you subtracted 2/3 to reach zero.

b) Redo the question, $4 \div 2/3$, by using a number line.

c) Should the answers for part a) and part b) be the same?

Question 2.

Obtain an answer to each fraction division using the FAST METHOD method. DO NOT reduce.

a) $4 \div 3/4$

b) $10 \div 2/5$

c) $2/3 \div 6$

Question 3.

Obtain an answer to each fraction division using the FAST METHOD. DO NOT reduce.

a) $6 \div 3/2$

b) $9 \div 3/8$

c) $6/7 \div 3$

Question 4.

Obtain an answer to each fraction division using the FAST METHOD. Reduce if possible.

a) $1/2 \div 3/4$

b) $2/3 \div 4/5$

c) $3/7 \div 3/10 \div 1/2$

Question 5.

Obtain an answer to each fraction division using the FAST METHOD. Reduce if possible.

a) $1\ 2/3 \div 3/8$

b) $4/5 \div 2\ 1/2$

c) $2\ 1/3 \div 3\ 2/5 \div 2/3$

Question 6.

Solve each of the following problems. Show your work.

a) Mark has 2 chocolate bars that he wants to share equally with friends. He cuts the bars up into 2/5 size pieces. How many friends will get a 2/5 size piece of chocolate bar?

b) Janice walks in order to keep fit. She walks a total of 3 7/10 km . If she walked 5 days in a row, what average distance did she walk in a day?

c) Barb had 16/20 of a litre of orange juice in the fridge. She needed to make 2/5 of a litre portions for breakfast. How many portions can she make?

d) Jason bought 2 3/5 kg of candies. He split the candy up into 1/5 of a kg portions. How many portions can he make?

Question 7.

Obtain an answer to each arithmetic task using an efficient method. Reduce if possible.

a) $1/2 + 1/3 + 10 \div 5/6$

b) $1/2 \div 3/4 + 3/2 - 6 \div 3/4$

c) $2\ 1/5 + 3/2 \div 2/5 - 1\ 3/10 \times 5/2$

MAINTAIN stage

Mini-task example

Every so often:

- Present two fraction division questions (fraction ÷ fraction and mixed ÷ fraction). Ask students to use the FAST METHOD for dividing fractions to obtain answers. Discuss as needed.

Rich-task example

Have students solve the following problem:

George has two bolts of red cloth that he wants to use to make bandanas. One bolt is 2 3/4 m long; the other is 1 1/2 m long. A bandana requires 1/4 m of cloth to make. How many bandanas can George make from the red cloth he has?

Comments

This is a rich-task because it is a complex problem involving fraction addition and division.